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# Reexamining the Constraint on the Helium Abundance from CMB

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## Abstract

We revisit the constraint on the primordial helium mass fraction  $Y_p$  from observations of cosmic microwave background (CMB) alone. By minimizing  $\chi^2$  of recent CMB experiments over 6 other cosmological parameters, we obtained rather weak constraints as  $0.17 \leq Y_p \leq 0.52$  at  $1\sigma$  C.L. for a particular data set. We also study the future constraint on cosmological parameters when we take account of the prediction of the standard big bang nucleosynthesis (BBN) theory as a prior on the helium mass fraction where  $Y_p$  can be fixed for a given energy density of baryon. We discuss the implications of the prediction of the standard BBN on the analysis of CMB.

# 1 Introduction

Recent precise cosmological observations such as WMAP [1] push us toward the era of so-called precision cosmology. In particular, the combination of the data from cosmic microwave background (CMB), large scale structure, type Ia supernovae and so on can severely constrain the cosmological parameters such as the energy density of baryon, cold dark matter and dark energy, the equation of state for dark energy, the Hubble parameter, the amplitude and the scale dependence of primordial fluctuation.

Among the various cosmological parameters, the primordial helium mass fraction  $Y_p$  is the one which has been mainly discussed in the context of big bang nucleosynthesis (BBN) but not that of CMB so far. One of the reason is that the primordial helium abundance has not been considered to be well constrained by observations of CMB since its effects on the CMB power spectrum is expected to be too small to be measured. However, since now we have very precise measurements of CMB, we may have a chance to constrain the primordial helium mass fraction from CMB observations. Since the primordial helium mass fraction can affect the number density of free electron in the course of the recombination history, the effects of  $Y_p$  can be imprinted on the power spectrum of CMB. Recently, some works along this line have been done by two different groups [2, 3], which have discussed the constraints on  $Y_p$  from current observations of CMB. In fact they claim different bounds on the primordial helium mass fraction, especially in terms of its uncertainty: the author of Ref. [2] obtained  $0.160 \leq Y_p \leq 0.501$ , on the other hand the authors of Ref. [3] got  $Y_p = 0.250^{+0.010}_{-0.014}$  at  $1\sigma$  confidence level. It should be noticed that the latter bound is much more severe than that of the former. If the helium mass fraction is severely constrained by CMB data, it means that the CMB power spectrum is sensitive to the values of  $Y_p$ . In such a case, the prior on  $Y_p$  should be important to constrain other cosmological parameters too and the usual fixing of  $Y_p = 0.24$  in CMB power spectrum calculations might not be a good assumption. Especially, analyses like Refs. [4, 5, 6, 7, 8] predict light element abundances including  $^4\text{He}$  from the baryon density which is obtained from the CMB data sets with the analysis fixing the value of  $Y_p$ . Such procedure is only valid when  $Y_p$  is not severely constrained by CMB. Thus it is very important to check the CMB bound on  $Y_p$ .

One of the main purpose of the present paper is that we revisit the constraint on  $Y_p$  from observations of CMB alone with a different analysis method from Markov chain Monte Carlo (MCMC) technique which is widely used for the determination of cosmological parameters and adopted in Refs. [2, 3]. In this paper, we calculate  $\chi^2$  minimum as a function of  $Y_p$  and derive constraints on  $Y_p$ . We adopt the Brent method of the successive parabolic interpolation to minimize  $\chi^2$  varying 6 other cosmological parameters of the  $\Lambda\text{CDM}$  model with the power-law adiabatic primordial fluctuation. We obtain the constraint on  $Y_p$  by this method and compare it with previously obtained results.

We also study the constraint on  $Y_p$  from future CMB experiment. A particular emphasis is placed on investigating the role of the standard BBN theory. Since the primordial helium is synthesized in BBN, once the baryon-to-photon ratio is given, the value of  $Y_p$  is fixed theoretically. Thus, using this relation between the baryon density and helium abun-

dance, we do not have to regard  $Y_p$  as an independent free parameter when we analyze CMB data. We study how the standard BBN assumption on  $Y_p$  affects the determination of other cosmological parameters in the future Planck experiment using the Fisher matrix analysis.

The structure of this paper is as follows. In the next section, we briefly discuss the effects of the helium mass fraction on the CMB power spectrum, in particular its effects on the change of the structure of the acoustic peaks. Then we study the constraint on the primordial helium mass fraction from current observations of CMB using the data from WMAP, CBI, ACBAR and BOOMERANG. In section 4, we discuss the expected constraint on  $Y_p$  from future CMB observation of Planck and also study how the standard BBN assumption on  $Y_p$  can affect the constraints on cosmological parameters. The final section is devoted to the summary of this paper.

## 2 Effects of the change of $Y_p$ on CMB

In this section, we briefly discuss the effects of the change of the helium abundance on the CMB power spectrum. More detailed description of this issue can be found in Ref. [2].

The main effect of  $Y_p$  on the CMB power spectrum comes from the diffusion damping at small scales. When  $Y_p$  is large, since it is easier for electrons to recombine with  $^4\text{He}$  than with H, the number of free electron becomes small. Thus the Compton mean free path becomes larger for larger  $Y_p$ , which means that the diffusion length of photon becomes also larger. Since the photon-baryon tight coupling breaks down at the photon diffusion scales, the fluctuation of photon is exponentially damped due to the diffusive mixing and rescattering. Hence the CMB power spectrum is more damped for larger values of  $Y_p$ . To see this tendency, we plot the CMB power spectra for several values of  $Y_p$  in Fig. 1. We clearly see that the damping at the small scales is more significant for the cases with larger values of  $Y_p$ . The effect of diffusion damping causes the change in the power spectrum at a percent level for 10 % change of  $Y_p$  [2].

To see this more quantitatively, we consider the ratio of the second peak height to the first which is defined as [9]

$$H_2 \equiv \left( \frac{\Delta T(l=l_2)}{\Delta T(l=l_1)} \right)^2, \quad (2.1)$$

and the third peak height to the first

$$H_3 \equiv \left( \frac{\Delta T(l=l_3)}{\Delta T(l=l_1)} \right)^2, \quad (2.2)$$

where  $(\Delta T(l))^2 \equiv l(l+1)C_l/2\pi$ . We do not discuss the first peak position and height because they are almost unaffected by the change in  $Y_p$ . We calculate the responses of these quantities with respect to the change in the cosmological parameters around the

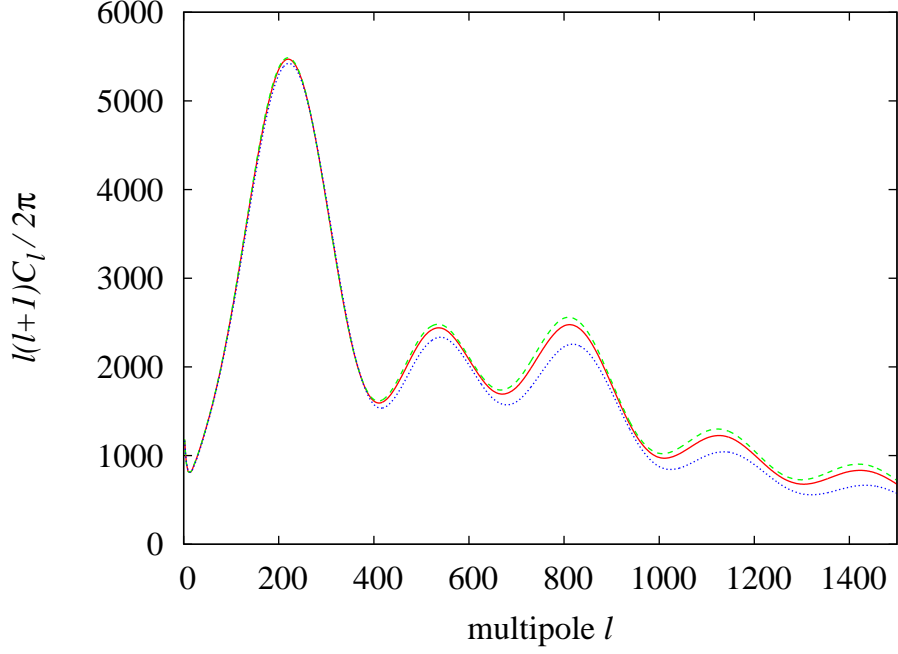


Figure 1: The CMB power spectra for the cases with  $Y_p = 0.1$  (green dashed line), 0.24 (red solid line) and 0.5 (blue dotted line). Other cosmological parameters are taken to be the WMAP mean values for the power-law  $\Lambda$ CDM model.

fiducial values,  $\omega_m = 0.14, \omega_b = 0.024, \Omega_\Lambda = 0.73, \tau = 0.166, n_s = 0.99$  and  $Y_p = 0.24$  where  $\omega_i \equiv \Omega_i h^2$  with  $\Omega_i$  being the energy density of component  $i$  normalized by the critical energy density. The subscript  $b$  denotes baryon and  $m$  stands for matter which is the sum of baryon and CDM.  $h$  is the Hubble parameter,  $\tau$  is the reionization optical depth and  $n_s$  is the scalar spectral index of primordial power spectrum. These values (except for  $Y_p$ ) are the mean values of WMAP for the power-law  $\Lambda$ CDM model [10]. Also, we keep the flatness of the universe when we change parameters. We found

$$\Delta H_2 = -0.30 \frac{\Delta \omega_b}{\omega_b} + 0.015 \frac{\Delta \omega_m}{\omega_m} + 0.41 \frac{\Delta n_s}{n_s} - 0.0125 \frac{\Delta Y_p}{Y_p}, \quad (2.3)$$

$$\Delta H_3 = -0.18 \frac{\Delta \omega_b}{\omega_b} + 0.21 \frac{\Delta \omega_m}{\omega_m} + 0.56 \frac{\Delta n_s}{n_s} - 0.029 \frac{\Delta Y_p}{Y_p}, \quad (2.4)$$

where we neglected the dependence on  $\Omega_\Lambda$  and  $\tau$  since their coefficients are very tiny even if compared to that of  $Y_p$ . From these expressions, we can see that the response of  $C_l$  to the change in  $Y_p$  is very sluggish. This is one of the reasons why we do not expect to obtain a meaningful constraint on  $Y_p$  from CMB until recently. Moreover, the change of  $C_l$  caused by varying  $Y_p$  is readily canceled by shifting other parameters. However, since observations of CMB now have become precise and cover wider multipole range, we may

have a chance to constrain  $Y_p$  from current observations of CMB, which will be discussed in the next section.

### 3 Constraint on $Y_p$ from current CMB observations

Now we study the constraint on the helium abundance from current observations of CMB. For this purpose, we use the data from WMAP [1], CBI [11] and ACBAR [12]. We also include the recent data from BOOMERANG experiment [13, 14, 15]. To calculate  $\chi^2$  from WMAP data, we used the code provided by WMAP [16, 17, 18]. For CBI, ACBAR and BOOMERANG, we made use of modules in COSMOMC [19]. As mentioned in the introduction, two groups have reported different bound on  $Y_p$  using CMB data alone, especially in terms of its uncertainties. One group has obtained  $0.160 \leq Y_p \leq 0.501$  [2] at  $1\sigma$  C.L., on the other hand the authors of Ref. [3] give the bound as  $Y_p = 0.250^{+0.010}_{-0.014}$ . The authors of Refs. [2] and [3] use the CMB data which cover the similar multipole region as that of ours. For details of their analysis, we refer the reader to Refs. [2] and [3]. If the severe bound on  $Y_p$  is obtained from current CMB data, it means that the CMB power spectrum is sensitive to the value of  $Y_p$  and the prior on  $Y_p$  would affect the constraints on other cosmological parameters. Thus it is important to check the bound independently.

For the analysis in this paper, we adopted a  $\chi^2$  minimization by nested grid search instead of Markov chain Monte Carlo (MCMC) method which was used in their analysis. In our analysis, we apply the Brent method [20] of the successive parabolic interpolation to find a minimum with respect to one specific parameter with other parameters at a given grid, then we iteratively repeat the procedure to find the global minimum. For the detailed description of this method, we refer the readers to Ref. [21]. Here we assume a flat universe and the cosmological constant for dark energy. We also assume no contribution from gravity wave. In Fig. 2, we show the values of  $\chi^2$  minimum as a function of  $Y_p$ . As seen from the figure, we do not have a severe constraint from current observations of CMB, which supports the result of Ref. [2]. Reading  $Y_p$  values which give  $\Delta\chi^2 = 1$ , we obtained the constraint at  $1\sigma$  C.L. as  $0.17 \leq Y_p \leq 0.52$  for the case where the data from WMAP, CBI and ACBAR are used. When the data from BOOMERANG is added, we got  $0.25 \leq Y_p \leq 0.54$ . We have also made the analysis for different data sets for comparison. In fact, we cannot obtain a significant constraint in the region  $0.1 < Y_p < 0.6$  using WMAP data alone. Even if we add the data from BOOMERANG, we cannot constrain the value of  $Y_p$ . Thus the data from CBI and ACBAR which cover high multipole regions are important to constrain  $Y_p$  although the constraint is rather weak.

As discussed in the previous section, the CMB power spectrum can be affected by changing the value of  $Y_p$ . However this change can be canceled by tuning other cosmological parameters to give almost the same CMB power spectra. To see this clearly, in Fig. 3, we show the CMB power spectra for several values of  $Y_p$  with other cosmological parameters being chosen to give almost indistinguishable angular power spectra. As seen from the figure, even if we take much larger or smaller values of  $Y_p$  than usually assumed, we can

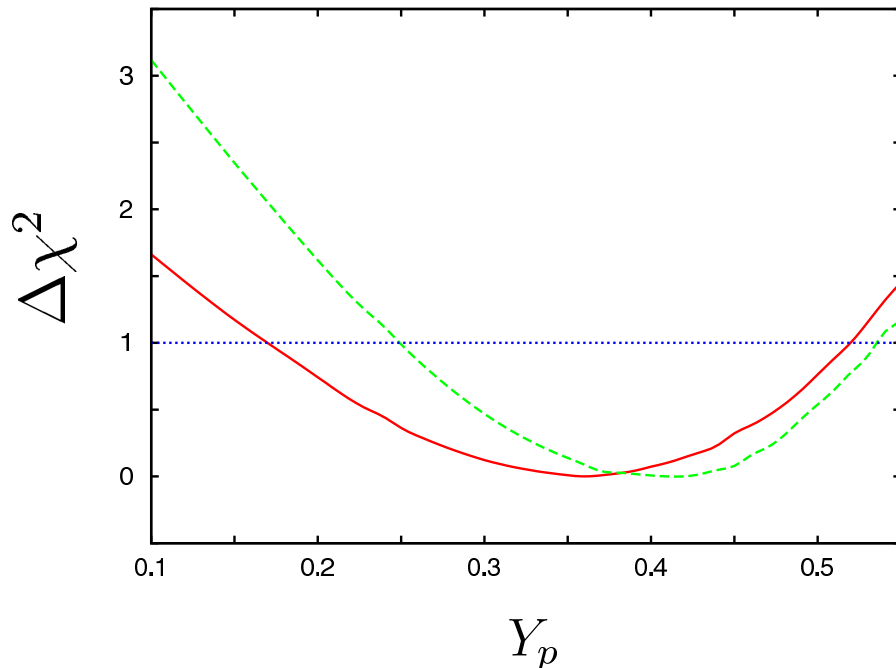


Figure 2: The values of  $\Delta\chi^2$  are shown as a function of  $Y_p$ . Other cosmological parameters are taken to minimize  $\chi^2$ . The red solid line is for the data of WMAP, CBI and ACBAR. The green dashed line includes BOOMERANG data in addition.

fit such values of  $Y_p$  to the data by tuning other cosmological parameters.

When we include the data from BOOMERANG, the favored values of  $Y_p$  are shifted to larger  $Y_p$ . Notice that higher multipoles are more suppressed by increasing  $Y_p$ , which is almost the same effect as decreasing  $n_s$ <sup>#1</sup>. Since BOOMERANG data favors red-tilted initial power spectrum compared to other data such as WMAP [22], it is reasonable that larger values of  $Y_p$  are favored by BOOMERANG. Particularly,  $Y_p = 0.24$  which is used in usual analysis is just out of the  $1\sigma$  bound. However, it is allowed at  $2\sigma$  C.L. so we do not take this as a serious discrepancy from the standard assumption.

## 4 Constraint on $Y_p$ from future CMB observations and the role of standard BBN theory

In this section, we discuss the future constraint on the primordial helium mass fraction and other cosmological parameters. We especially want to investigate how the constraints are modified when we take account of the relation between  $\omega_b$  and  $Y_p$  fixed by the standard

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<sup>#1</sup>Eqs. (2.3) and (2.4) show these properties quantitatively. We remark that they are also useful to understand the tendency that the values of  $n_s$  which give the minimum  $\chi^2$  for fixed  $Y_p$  become larger as we increase  $Y_p$ . This is because the suppression of higher multipoles caused by increasing  $Y_p$  can be compensated by increasing  $n_s$ .

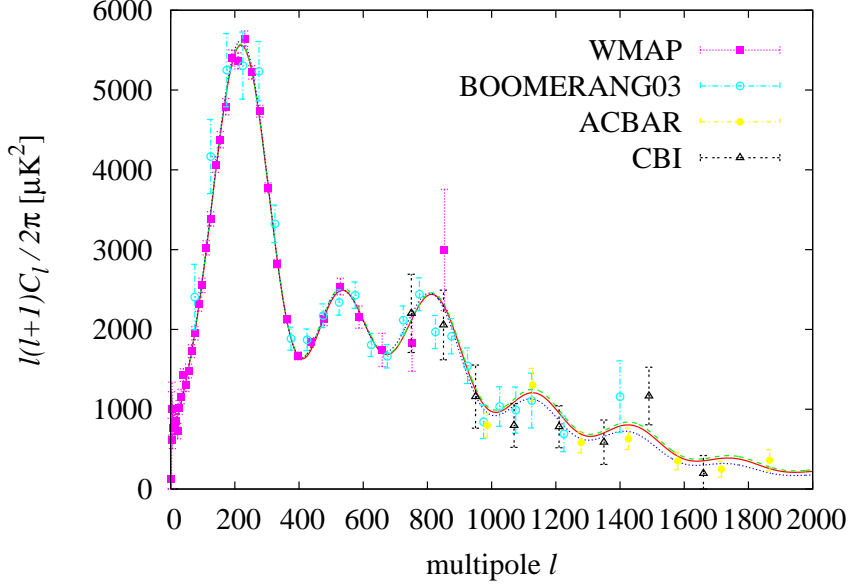


Figure 3: The CMB power spectra for the case with  $Y_p = 0.24, \omega_m = 0.135, \omega_b = 0.023, h = 0.72, \tau = 0.117, n_s = 0.96$  (red solid line),  $Y_p = 0.1, \omega_m = 0.130, \omega_b = 0.023, h = 0.72, \tau = 0.101, n_s = 0.95$  (green dashed line) and  $Y_p = 0.5, \omega_m = 0.148, \omega_b = 0.023, h = 0.72, \tau = 0.117, n_s = 0.99$  (blue dotted line). Notice that these power spectra are almost indistinguishable up to multipole region  $l \sim 1000$ .

BBN theory. As mentioned in the introduction, the CMB anisotropies can be measured more precisely in the future, thus the primordial helium mass fraction may well be determined from CMB observations alone. The future constraints on  $Y_p$  has already been investigated in Ref. [2] using the expected WMAP 4 year data, Planck and cosmic variance limited experiments. Here we study this issue supplementing the consideration regarding the prediction of the BBN theory.

When we only consider observations of CMB alone, the primordial helium mass fraction  $Y_p$  can be viewed as one of free independent parameters. However, when we take account of the BBN theory,  $Y_p$  is not an independent parameter any more but is related to the value of the baryon-to-photon ratio or the energy density of baryon. Below, we discuss how such relation derived from the BBN calculation affects the determination of cosmological parameters in the future Planck experiment.

First we give the relation between  $Y_p$  and the baryon-to-photon ratio  $\eta$  from the calculation of the standard BBN.  $\eta$  and the baryon density  $\omega_b$  are related as  $10^{10}\eta = 273.49\omega_b$ . Some groups have reported the fitting formula for  $Y_p$  as a function of  $\eta$  [8, 23, 24, 25].

Here we adopt the fitting formula given in Ref. [8]

$$10Y_p = \left[ \sum_{n=1}^8 a_n x^{n-1} + \sum_{n=1}^8 b_n x^{n-1} \Delta N + \sum_{n=1}^8 c_n x^{n-1} (\Delta N)^2 + \sum_{n=1}^8 d_n x^{n-1} (\Delta N)^3 \right] \times \exp \left( \sum_{n=1}^6 e_n x^n \right), \quad (4.5)$$

where  $x \equiv \log_{10}(10^{10}\eta)$ , the coefficients  $a_n, b_n, c_n, d_n$  and  $e_n$  are given in Ref. [8] and  $\Delta N$  represents the number of effective degrees of freedom of extra relativistic particle species. The standard BBN case is obtained with  $\Delta N = 0$ . According to Ref. [8], the accuracy of this formula is better than 0.05 % for the range of  $5.48 \times 10^{-10} < \eta < 7.12 \times 10^{-10}$  ( $0.02 < \omega_b < 0.026$ ) which corresponds to the  $3\sigma$  range obtained from WMAP and  $-3 < \Delta N < 3$ . Since scenarios with  $\Delta N \neq 0$  have been discussed in the literature including the possibility of negative  $\Delta N$  such as dark radiation in brane world scenario, varying gravitational constant and so on<sup>#2</sup>, we consider two cases when we discuss the future constraints. The first one is the case of the standard BBN, in other words, we assume that the energy density of extra radiation component as a fixed parameter with  $\Delta N = 0$ . For the other case, we treat  $\Delta N$  as a usual cosmological parameter which we vary, namely we assume  $\Delta N \neq 0$ . In this case, we use Eq. (4.5) to obtain  $Y_p$  for given  $\omega_b$  and  $\Delta N$ .

Now we discuss the expected constraint from future CMB observations. For this purpose, we adopt the Fisher matrix method. Thus first we briefly review the Fisher matrix analysis which is widely used in the literature to study the future constraints on cosmological parameters. Detailed descriptions of this analysis method can be found in Refs. [30, 31, 32, 33]. For the CMB data, the Fisher matrix can be written as

$$F_{ij} = \sum_l \sum_{X,Y} \frac{\partial C_l^X}{\partial x_i} \text{Cov}^{-1}(C_l^X, C_l^Y) \frac{\partial C_l^Y}{\partial x_j}, \quad (4.6)$$

where  $X, Y = TT, TE, EE$ ,  $x_i$  represents a cosmological parameter and  $\text{Cov}(C_l^X, C_l^Y)$  is the covariance matrix of the estimator of the corresponding CMB power spectrum which is given explicitly in Ref. [31]. The  $1\sigma$  uncertainty can be estimated as  $\sqrt{(F^{-1})_{ii}}$  for a cosmological parameter  $x_i$ . For the fiducial model, we assumed the cosmological parameters as  $A = 0.86$ ,  $\omega_m = 0.14$ ,  $\omega_b = 0.024$ ,  $\Omega_\Lambda = 0.73$ ,  $\tau = 0.166$  and  $n_s = 0.99$ . Here,  $A$  represents the amplitude of scalar perturbation whose normalization is taken to be same as that of WMAP team [10]. The fiducial value for  $Y_p$  is fixed using the BBN relation Eq. (4.5) for a given  $\omega_b$  and  $\Delta N$  unless otherwise stated. A flat universe is assumed and we do not consider the contribution from the tensor mode. For dark energy, we assumed

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<sup>#2</sup>The negative values of  $\Delta N$  can also arise in a scenario with low reheating temperature  $T_{\text{ref}} \sim \mathcal{O}(\text{MeV})$ . However, in this kind of scenario, the neutrino distribution functions are deviated from the thermal ones so the primordial helium abundance is modified in a way that the fitting formula Eq. (4.5) does not apply [26, 27, 28, 29]. The most recent  $Y_p$  calculation in the low reheating scenario including the effects of neutrino oscillations are given in Ref. [29].



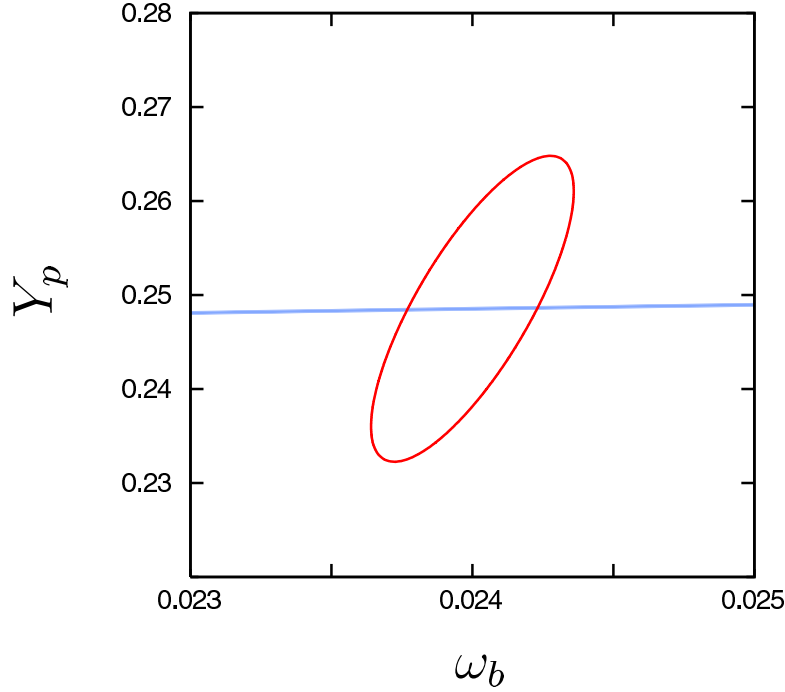


Figure 4: Expected contour of the  $1\sigma$  constraint from Planck experiment. We take  $Y_p$  as an independent free parameter. Other cosmological parameters are marginalized over in this figure. The thin and nearly horizontal band is the theoretical BBN calculation of  $Y_p$  as a function of  $\omega_b$  with its width representing  $1\sigma$  error from reaction rates. Here we assumed  $\Delta N = 0$ .

the cosmological constant. To forecast uncertainties, we use the future data from Planck [34] whose expected instrumental specifications can be found in Ref. [33].

Now we show our results. First we present the case with  $Y_p$  being treated as a free parameter, namely we do not consider the BBN relation. In Fig. 4, we show the expected  $1\sigma$  contour in the  $\omega_b$  vs.  $Y_p$  plane. Other cosmological parameters are marginalized. We also draw the band for  $Y_p$  as a function of  $\omega_b$  from theoretical calculation of the standard BBN with  $1\sigma$  error due to the uncertainties in the reaction rates. The uncertainty is dominated by that of the neutron lifetime, which is very small. As is clearly seen from the figure, the constraint on  $Y_p$  from Planck is not significant compared to the uncertainties of the standard BBN calculation. Hence as far as we take account of the standard BBN, the helium mass fraction can be fixed using the BBN relation of Eq. (4.5) even for the precision measurements of CMB such as Planck.

Next we discuss how the theoretical BBN relation can affect the determinations of cosmological parameters. First we consider the case with  $\Delta N = 0$ . When we determine the value of  $Y_p$  for a given  $\omega_b$  using Eq. (4.5) (to be more specific, when we calculate numerical derivatives with respect to  $\omega_b$ , we simultaneously varied  $Y_p$  following Eq. (4.5))

	$A$	$\omega_b$	$\omega_m$	$\Omega_\Lambda$	$\tau$	$n_s$	$Y_p$	$\Delta N$
w/o BBN rel. ( $\Delta N = 0$ )	0.132	0.00068	0.0033	0.025	0.067	0.025	0.030	—
w/ BBN rel. ( $\Delta N = 0$ )	0.071	0.00022	0.0018	0.010	0.036	0.005	—	—
w/o BBN rel. ( $\Delta N \neq 0$ )	0.164	0.00098	0.0041	0.041	0.086	0.038	0.031	0.41
w/ BBN rel. ( $\Delta N \neq 0$ )	0.120	0.00074	0.0031	0.035	0.065	0.029	—	0.41

Table 1: Expected  $1\sigma$  uncertainties from Planck experiment using the temperature fluctuation alone. See the text for the fiducial values used in the analysis.

	$A$	$\omega_b$	$\omega_m$	$\Omega_\Lambda$	$\tau$	$n_s$	$Y_p$	$\Delta N$
w/o BBN rel. ( $\Delta N = 0$ )	0.0106	0.00024	0.0012	0.0081	0.0054	0.0075	0.011	—
w/ BBN rel. ( $\Delta N = 0$ )	0.0097	0.00015	0.0012	0.0067	0.0052	0.0035	—	—
w/o BBN rel. ( $\Delta N \neq 0$ )	0.0107	0.00025	0.0029	0.0090	0.0055	0.0082	0.014	0.19
w/ BBN rel. ( $\Delta N \neq 0$ )	0.0099	0.00023	0.0020	0.0091	0.0054	0.0078	—	0.14

Table 2: Expected  $1\sigma$  uncertainties from Planck experiment using both the temperature and polarization data. See the text for the fiducial values used in the analysis.

we can expect that the uncertainties of other cosmological parameters are reduced to some extent. In Tables 1 and 2, we show the uncertainties of cosmological parameters from the future Planck experiment using the information of TT spectrum alone and that including polarization spectrum, respectively. The first and second rows in the tables correspond to the case without and with the BBN relation. For these cases, we assumed  $\Delta N = 0$ . Furthermore, we also show the cases with  $\Delta N \neq 0$  in the third and forth row in the tables.

Now we discuss the cases with  $\Delta N = 0$ . As seen from the tables, when we assume the BBN relation, the uncertainties become smaller by a factor of  $\mathcal{O}(1)$  compared to that for the case with  $Y_p$  being an independent free parameter. The parameter which receives the benefit most is  $n_s$ . This is consistent with the fact that this parameter is the most degenerate parameter with  $Y_p$ . Meanwhile, we note that the value of  $Y_p$  fixed by the BBN relation for  $\omega_b = 0.024$  is slightly different from  $Y_p = 0.24$  which is usually used in the literature. We also evaluated the uncertainties fixing the helium mass fraction as  $Y_p = 0.24$  independent of  $\omega_b$  and checked that the  $1\sigma$  errors are quite similar to those for the case with  $Y_p$  being related to  $\omega_b$  by Eq. (4.5). Since the change of  $C_l$  with respect to that of  $Y_p$  is very small, in other words the derivative of  $C_l$  with respect to  $Y_p$  is very small, we can have almost the same result even if we use a slightly different value for  $Y_p$ . Thus we can just fix the value of  $Y_p$  instead of using Eq. (4.5) even for the future CMB experiments such as Planck.

Here the discussion for the case with  $\Delta N \neq 0$  is in order. Here we treat  $\Delta N$  as one of the cosmological parameters which should be varied. Notice that the addition of an extra radiation component can affect the CMB power spectrum through the speed up of the Hubble expansion and the early ISW effect because of the change of the radiation-matter

equality epoch. In fact, some authors have discussed the future constraints on cosmological parameters paying attention to  $\Delta N$  [35, 36] but without considering the effect of  $Y_p$ . Here we study this issue allowing  $Y_p$  to vary and also investigate the implications of the BBN relation on the future constraints on them. For the purpose of the Fisher matrix analysis, we assumed  $\Delta N = 0$  as the fiducial value. As already discussed,  $Y_p, \omega_b$  and  $\Delta N$  are related by Eq. (4.5) from the BBN calculation. In Fig. 5, we show the expected  $1\sigma$  contour in the  $\omega_b$  vs.  $\Delta N$  plane from Planck experiment for the cases with and without assuming the BBN relation. Naturally, when we assume the BBN relation, the uncertainties become smaller because it reduces the number of independent parameters. Also notice that, since the BBN theory relates  $\omega_b$  with  $\Delta N$ , the contour shrinks to the direction of correlation between  $\omega_b$  and  $\Delta N$ .

In the third and forth rows of Tables 1 and 2, the uncertainties of cosmological parameters are also shown for the cases with and without the BBN relation being imposed among  $Y_p, \omega_b$  and  $\Delta N$  respectively. As expected, the uncertainties for cosmological parameters for the case with the BBN relation are smaller, however the differences are not so large.

Here we comment on the implications of the BBN relation on the constraints from current observations of CMB. We have also made the analysis adopting the BBN relation to fix the value of  $Y_p$  for given  $\omega_b$ . The constraint on  $\omega_b$  for this case is almost unchanged compared to the case with  $Y_p = 0.24$  being fixed. This can be more or less expected from the result we have shown in the previous section. This again shows that current observations of CMB are not so sensitive to the values of  $Y_p$ . Thus predicting the helium abundance by the BBN theory using the CMB value of  $\omega_b$ , namely the procedure adopted in Refs. [4, 5, 6, 7, 8], is valid at least with the current quality of the CMB data.

## 5 Summary

We revisited the constraint on the primordial helium mass fraction  $Y_p$  from current observations of CMB. Some authors have already studied the constraint [2, 3], however their results were different especially in terms of the uncertainty. One of the main purpose of the present paper is to study the constraints on  $Y_p$  from current observations adopting a different analysis method. Instead of MCMC method which was adopted by the authors of Refs. [2, 3] to obtain the constraint, here we adopted a  $\chi^2$  minimization by a nested grid search. We did not obtain a severe constraint in agreement with Ref. [2]. Using the data from WMAP, CBI and ACBAR as well as recent BOOMERANG data, we get  $1\sigma$  constraint as  $0.25 \leq Y_p \leq 0.54$  and  $0.17 \leq Y_p \leq 0.52$  for the cases with and without the data from BOOMERANG. It might be interesting to note that usual assumption of  $Y_p = 0.24$  is not in the  $1\sigma$  error range of BOOMERANG combined analysis but it is not of high significance at this stage so we can safely assume  $Y_p = 0.24$  for current CMB data analysis.

We also studied the future constraint from CMB on  $Y_p$  taking account of the standard BBN prediction as a prior on  $Y_p$ . Although we cannot obtain a severe constraint

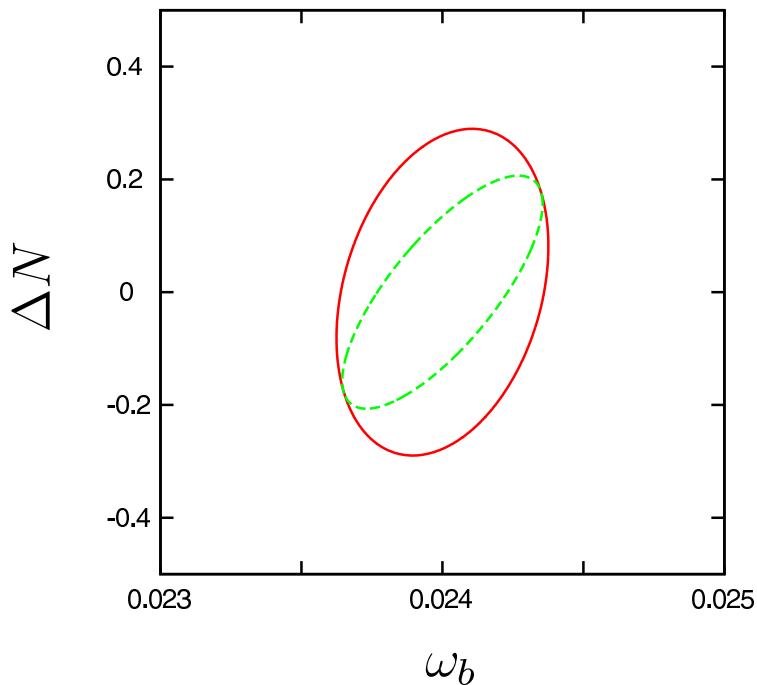


Figure 5: Expected contours of  $1\sigma$  constraints from Planck experiment for the cases without the BBN relation (solid red line) and with it (dashed green line). We marginalized over other cosmological parameters in this figure.  $Y_p$  is determined by the BBN relation.

at present, observations of CMB can be much more precise in the future. Thus we may have a chance to obtain a precise measurement of  $Y_p$  from upcoming CMB experiments. On the other hand, since the primordial helium has been formed during the time of BBN, once the baryon-to-photon ratio is given, the value of  $Y_p$  can be evaluated theoretically assuming the standard BBN. Thus, in this case, we do not have to assume  $Y_p$  as an independent free parameter when we analyze CMB data. We studied how such BBN theory prior on  $Y_p$  affects the determination of other cosmological parameters in the future Planck experiment. We evaluated the uncertainties for the case with  $Y_p$  being an independent free parameter and  $Y_p$  being fixed for a given  $\omega_b$  using the BBN relation. We showed that the BBN prior improves the constraints on other cosmological parameters by a factor of  $\mathcal{O}(1)$  and also it induces some correlations among the parameters which appear in the BBN relation. As shown in Fig. 4, as far as we consider the standard scenario of cosmology, the helium mass fraction can be fixed for CMB analysis even in the future experiments since we can expect that the constraint from Planck is much weaker than the uncertainty of the theoretical calculation of the standard BBN. However, it is worthwhile to do CMB analysis treating  $Y_p$  as a free parameter and measure the helium mass fraction independently from the baryon density since it provides a consistency test for the standard BBN theory (of course, measurements of primordial light element abundances by astrophysical means provide further consistency tests). By checking the robustness of the consistency

from various observations, the golden age of precision cosmology can push us toward the accurate understanding of the universe.

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